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FLEXURAL ANALYSIS OF SMART STRUCTURAL COMPOSITE LAMINATES BY USING A NEW HIGHER ORDER THEORY

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ABSTRACT

The present article involved the enrichment of a higher order shear deformation theory especially for composite laminates embedded with piezoelectric materials under electro-mechanical loading. The main aim of the present investigation is to propose the analytical formulations and solutions to examine the flexural behavior of the laminated composite plates embedded with piezoelectric fiber reinforced composite by using a higher order theory. The assumed two models M1 & M2 are developed separately with individual higher order theories. The principle of virtual work is used in order to obtain the boundary conditions and the governing equations of equilibrium. The Navier's technique is involved in order to obtain the solutions. The results are tabulated and analyzed for different aspect ratios, voltages for the study of variation of in plane and transverse displacements and also the normal stresses and transverse shear stresses.

KEYWORDS: Smart Materials, Higher Order Shear Deformation Theory, Principle of Virtual Work & Navier's Technique

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Original Article

1. INTRODUCTION

The conventional structures have no control over the induced deformation where as in smart structures the state of deformation is continuously monitored and has good control over its configuration of deformation. The latest development in the composite science is the piezoelecteic materials are embedded to them in form of distributed sensors or actuators. It is really a good focus on the development of innovative materials in the field of material science engineering. Combining the properties of conventional composites with the intelligent materials like piezoelectric materials, bimetal alloys etc. results the generations of smart composite materials. The advanced engineering applications like air craft, aerospace, naval structures, rocket and missile technology, defense and war appliances etc. are always needed the smart or intelligent structural materials in their fabrication.

Laminated composite structures, with embedded piezoelectric actuators and sensors combines some of the superior mechanical properties of composites with the additional capabilities to sense deformations and stress states and to adapt their response accordingly. Piezo-laminates as smart-intelligent composites offer great potential for active control of advanced aerospace, nuclear, and automotive structural applications. Kant and Swaminathan [2002] are presented analytical solutions for the static analysis of laminated composite and sandwich plates based on a higher order refined theory. Shiyekar S. M., Tarun Kant [2011] are cosnsidered the higher order shear deformation effects on analysis of laminates with piezoelectric fiber reinforced composite actuators**.** P. Hemavathi, P. Veera Sanjeeva Kumar [2017] has investigated the Bending Analysis of Smart Composite Laminated Plates Subjected To Electro-Mechanical Loading Using HOSDT". Raju P.R., J. Suresh Kumar [2014] are presented a

paper on the Buckling analysis of smart material plates using higher order theory. P.V.S. Kumar, B.C.M. Reddy [2016] are submitted an article on the Bending analysis of smart composite laminated plates using higher order theory. A.M.A. Neves, A.J.M. Ferreira [2016] are executed the Free vibration and buckling analysis of laminated plates by oscillatory radial basis functions. Yunying Zhou, Jun Zhu [2016] are presented a paper to analyze the Vibration and bending analysis of multiferroic rectangular plates using third order shear deformation theory. P.V.S. Kumar, B.C.M. Reddy, K.V.K. Reddy [2016] are submitted an article to pursue the Transient analysis of smart composite laminated plates using a higher order theory. Reddy J.N., [1999] has discussed in his paper regarding the analysis by using the higher order theory on laminated composite plates with integrated sensors and actuators. Rajan L. Wankhade, Kamal M. Bajoria [2016] is submitted a paper on Shape control and vibration analysis of piezolaminate plates subjected to electro-mechanical loading. B. Sidda Reddy, J. Suresh Kumar C. Eswar Reddy [2014] are presented a paper on Free vibration behavior of functionally graded plates using higher order shear deformation theory. Manjunath BS., and T. Kant [1993] are proposed about the New theories for symmetric/unsymmetrical composite and sandwich beams with C^0 finite elements. P.V.S. Kumar, B.C.M. Reddy [2017] are performed vibration analysis of smart composite laminated plates using a higher order theory in their presented paper.. Mallik N., MC. Ray [2004] are found the exact solutions for analysis of piezoelectric fiber reinforced composites as a distributed actuators for smart composite laminate plates. P. Ravikant, J. Suresh Kumar [2014] are presented a paper on Bending analysis of smart material plates using higher order theory. J.L. Mantari, A.S. Oktem, C.G. Soares [2012] are proposed a new higher orders shear deformation theory for sandwich and composite laminated plates. Robbins D H., and J.N. Reddy [1999] have done an analysis of piezoelectrically actuated beams using a layer wise displacement theory in their presented article. In the present investigation at very first time proposed a higher order theory for two assumed models M1 & M2 with '9' and '12' unknown variables. The theoretical formulation and the analytical solutions are developed for symmetric and antisymmetric cross ply laminates. The Navier's tequnique is used to fin the analytical solutions and the results are obtained to investigate the effect of aspect ratio and electrical voltages on the induced in plane and transverse displacements and also on the normal and transverse shear stresses.

2. THEORETICAL FORMULATION

A rectangular cross ply laminated plate provided with simply supported condition is to be taken for present flexural analysis as it depicted in Figure 1. The top and bottom surfaces of the plate is embedded with the thin patches or layers of piezoelectric fiber reinforced composite material (PFRC). The attached material is functioning as the distributed sensor or actuator. The coordinate axes x,y and z are taken with reference to the middle layer of the laminate and the' z' axis direction is in the thickness direction of the laminate and it is equidistance from the top and bottom of the substrate. The thickness of the PFRC actuator is assumed as $\mathbf{t_p}$. The displacement vectors u(x,y,z), v(x,y,z) and w(x,y,z) at any point in the laminate are expanded in the powers of 'z' axis and are expanded in the following form of equation for two assumed models M1 & M2.

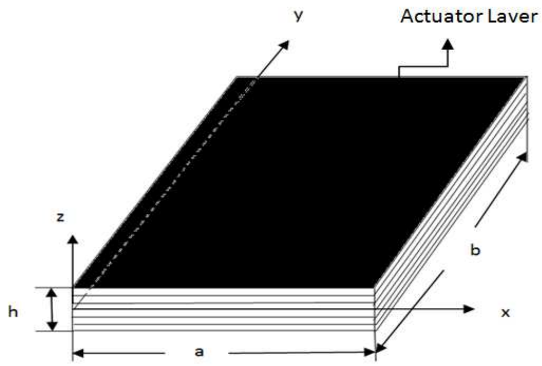

Figure 1: Geometry of Laminated Composite Plate with Simply Supported All Edges with PFRC Actuator at Top

Model-1(M1)

$$\left.\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2 u_o^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2 v_o^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_o(x,y) \end{aligned}\right\}$$

Model-2(M2)

$$\left.\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2 u_o^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2 v_o^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_o(x,y) + z\theta_z(x,y) + z^2 w_o^*(x,y) + z^3\theta_z^*(x,y) \end{aligned}\right\} \tag{1}$$

Where the parameters $u_{0,}$ v_0 denotes the in plane displacements and w_0 is the transverse displacement at any point on the mid plane of the substrate. The terms $\theta x, \theta y, \theta z$ are the rotations of the normal to the mid plane about y and x –axes respectively. The remaining terms u_0^*, v_0^*, w_0^*, θ_x^*, θ_y^*, and θ_z^* are the related higher–order deformation terms and they represent higher–order transverse cross sectional deformation modes.

2.1. Strain-Displacement Relations

Considering the Model-1 for the present analysis and by substitution of the displacement relations in Equation (1) in to the strain displacement equations of the classical theory of elasticity, the following relations are obtained:

$$\begin{aligned} \varepsilon_x &= \varepsilon_{xo} + zk_x + z^2\varepsilon_{xo}^* + z^3 k_x^* \\ \varepsilon_y &= \varepsilon_{yo} + zk_y + z^2\varepsilon_{yo}^* + z^3 k_y^* \\ \varepsilon_z &= 0 \end{aligned}$$

$$\begin{aligned} \gamma_{xy} &= \varepsilon_{xyo} + zk_{xy} + z^2\varepsilon_{xyo}^* + z^3 k_{xy}^* \\ \gamma_{yz} &= \phi_y + z\varepsilon_{yzo} + z^2\phi_y^* \\ \gamma_{xz} &= \phi_x + z\varepsilon_{xzo} + z^2\phi_x^* \end{aligned}$$

2.2. Lamina Coupled Constitutive Equations

The linear constitutive relations for a single piezoelectric layer couples the elastic and electric fields as given below,

The constitutive relations for a single piezoelectric layer pairs with the elastic and electric fields is given as

$\{\sigma\} = [Q]\,\{\varepsilon\} - [e]\,\{E\}$,

$\{D\} = [e]^t\{\varepsilon\} - [\eta]\,\{E\}$. (2)

Also the elastic field intensity vector 'E' in connection to the electrostatic potential ξ(x,y,z) at the Lth layer is represented as:

$$E_X^L = \frac{\partial\xi(x,y,z)}{\partial x};\quad E_y^L = \frac{\partial\xi(x,y,z)}{\partial y}\quad ;\quad E_z^L = \frac{\partial\xi(x,y,z)}{\partial z} \tag{3}$$

Where $\boldsymbol{\sigma}, \boldsymbol{Q}, \boldsymbol{\varepsilon}, \boldsymbol{e}, \boldsymbol{E}, \boldsymbol{D}$ and $\boldsymbol{\eta}$ are the stress vector, elastic constant matrix, strain vector, piezoelectric constant matrix, electric field intensity vector, electric displacement vector and dielectric constant matrix respectively. From the lamina coupled constitutive equations, the elastic field can be written as two components of stresses. The first is elastic stress component (es) and second is piezoelectric stress component (pz).

i.e $\{\sigma\} = \{\sigma\}^{es} - \{\sigma\}^{pz}$, (4)

2.3. Admissible Boundary Conditions for the Navier's Solutions of the Displacement Model

The present higher order displacement model is further reformed by using the governing equations of motion expressed in terms of the displacements in order to utilize the benefits of Navier's technique and its solutions. The obtained displacements are usually expanded in terms of unknown parameters in the Navier's technique. The choice of the trigonometric functions in the series is restricted to those which satisfy the boundary conditions of the problem. For the present work a rectangular laminated plate is considered with two sets of simply supported boundary conditions and the Navier's solutions are developed for it. The mechanical and electrical in plane boundary conditions for the assumed model is expressed as:

At edges x=0 and x=a; $v_0=0$, $w_0=0$, $\theta y=0$, $\theta z=0$, Mx=0, Nx=0, $v_0^*=0$, $w_o^*=0$, $\theta_y^*=0$, $M_x^*=0$, $N_x^*=0$, $\xi=0$

At edges y=0 and y=b; $u_0=0$, $w_0=0$, $\theta x=0$, $\theta_z=0$, $M_y=0$, $N_y=0$, $u_0^*=0$, $w_o^*=0$, $\theta_x^*=0$, $M_y^*=0$, $N_y^*=0$, $\xi=0$

The governing equations of higher-order theory for the obtained stresses and strains will be derived using the dynamic version of the principle of virtual displacements, i.e.

$$\int_0^T (\delta U + \delta V - \delta K)\, dt = 0$$

The true strains are determined by the true displacements by the same way of methodology the virtual strains are to be obtained with the virtual displacements.

By Substituting the virtual strains in the governing equation of motion and integrating by-parts to relieve the virtual generalized displacements δu_o, δv_0, δw_0, $\delta\theta_x$, $\delta\theta_y$, δu_0^*, δv_0^*, $\delta\theta_x^*$, $\delta\theta_y^*$ in a domain of any differentiation the

statement of virtual work.

Integrating the resulting resultant equation by parts and collecting coefficients of each of virtual displacements δu_o, δv_0, δw_0, $\delta\theta_x$, $\delta\theta_y$, δu_0^*, δv_0^*, $\delta\theta_x^*$, $\delta\theta_y^*$ together and noting that the virtual displacements are zero, the following ‘ 9’ equations of motion are obtained:

$$\delta u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_1 \ddot{u}_0 + I_2 \ddot{\theta}_x + I_3 \ddot{u}_0^* + I_4 \ddot{\theta}_x^*$$

$$\delta v_0 : \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = I_1 \ddot{v}_0 + I_2 \ddot{\theta}_y + I_3 \ddot{v}_0^* + I_4 \ddot{\theta}_y^*$$

$$\delta w_0 : \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = I_1 \ddot{w}_0$$

$$\delta\theta_x : \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = I_2 \ddot{u}_0 + I_3 \ddot{\theta}_x + I_4 \ddot{u}_0^* + I_5 \ddot{\theta}_x^*$$

$$\delta\theta_y : \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = I_2 \ddot{v}_0 + I_3 \ddot{\theta}_y + I_4 \ddot{v}_0^* + I_5 \ddot{\theta}_y^*$$

$$\delta u_0^* : \frac{\partial N_x^*}{\partial x} + \frac{\partial N_{xy}^*}{\partial y} = I_3 \ddot{u}_0 + I_4 \ddot{\theta}_x + I_5 \ddot{u}_0^* + I_6 \ddot{\theta}_x^*$$

$$\delta v_0^* : \frac{\partial N_y^*}{\partial y} + \frac{\partial N_{xy}^*}{\partial x} = I_3 \ddot{v}_0 + I_4 \ddot{\theta}_y + I_5 \ddot{v}_0^* + I_6 \ddot{\theta}_y^*$$

$$\delta\theta_x^* : \frac{\partial M_x^*}{\partial x} + \frac{\partial M_{xy}^*}{\partial y} - Q_x^* = I_4 \ddot{u}_0 + I_5 \ddot{\theta}_x + I_6 \ddot{u}_0^* + I_7 \ddot{\theta}_x^*$$

$$\delta\theta_y^* : \frac{\partial M_y^*}{\partial y} + \frac{\partial M_{xy}^*}{\partial x} - Q_y^* = I_4 \ddot{v}_0 + I_5 \ddot{\theta}_y + I_6 \ddot{v}_0^* + I_7 \ddot{\theta}_y^* \qquad (5)$$

By the same way of methodology the Model-2 also attempted and ‘12’ generalized equations of motion are obtained.

The Navier’s technique is used and considering the assumed boundary conditions, the mechanical load, electrical load then the related obtained mid plane displacements are expanded in Fourier series manner. The obtained equation of motion are used to determine the various in plane, normal and shear stresses.

3. RESULTS AND DISCUSSIONS

In the present work, various numerical examples are presented and discussed for verifying the accuracy and efficiency of the present higher order theory in predicting the maximum center deflections and various stresses of the simply supported unidirectional composite plate subjected to a sinusoidal transverse load. These deflections and stresses are obtained for various aspect ratios (S=10, S=20 and S=100), thickness coordinates, and voltages with and without piezo

effects. The values of maximum center deflection and stresses are obtained from constitutive relations. The results obtained through this work are good agreement with 3D elastic solutions, compared with other theories in the literature. The following material properties are used for unidirectional laminate composite plates.

The nondimensionalized maximum normal stresses, shear stresses and displacements for various aspect ratios and voltages are tabulated in table 1 & table 2 for symmetric and anti symmetric laminated composite plates with and without piezoelectric effect for model-1 respectively. The numerical values obtained from the theory developed are compared with all other theories available in the literature.

Elastic Layer (Graphite/Epoxy)

$E_2 = E_3 = 10^6$ Gpa; $E_1/E_2 = 25$; $G_{12} = 0.5\, E_2$; $G_{23} = 0.2\square_2$ $G_{12} = G_{13} = 0.5$;

$\nu 12 = \nu 23 = \nu 13 = 0.25$;

PFRC (Piezoelectric Fiber Reinforced Composite)

C_{11}=32.6 Gpa, $C_{12}=C_{21}$=4.3 Gpa ; $C_{13}=C_{31}$=4.76 Gpa ; $C_{22}=C_{33}$=7.2 Gpa; C_{23}=3.85Gpa;

C_{44}=1.05 Gpa; $C_{55}=C_{66}$=1.29 Gpa; e_{31}= -6.76 C/m^2;

$\eta_{11} = \eta_{22}$ = 0.037 E-9 C/Vm; η_{33} =10.64 E -9 C/Vm;

The transverse displacement, normal stresses and shear stresses are presented in nondimensionalized form using the following multipliers

$$m_1 = \frac{\sigma_x}{q_0 S^2}\ ,\ m_2 = \frac{\sigma_z}{q_0}\ ,\ m_3 = \frac{\tau_{zz}}{q_0}\ ,\ m_4 = \frac{100 E_T}{q_0 S^4 H}\ (w)$$

Figure 2 shows the variation of non-dimensionalized transverse displacement (u) for various aspect ratios and for voltages with and without the piezoelectric actuator is observed. It is observed from the figure that the displacement is drastically decreased for aspect ratio from 20 to 40 at 100V. It is also observed that for zero voltage the variation in the displacements is negligible irrespective of the aspect ratio. The cause may be due to the thick plate. The variation of non-dimensionalized shear stress (τ_{xz}) for various span to aspect ratio with and without piezo actuator are shown in Figure 3 for non piezoelectric unidirectional laminate subjected to mechanical loading. From the figure it is observed that the maximum shear stresses are found at a/h=40 and it is drastically decreases to a/h= 20, and again slightly increases and then it becomes constant between the aspect ratio of 20 to 10. The effect of transverse shear stress for with and without piezo actuator is very close. The variation of non-dimensional shear stresses (τ_{xy}) for aspect ratio with and without piezo actuator is shown in Figure 4. It is monitored that the nature of the stress is changing form compressive to tensile, near along the aspect ratio 20 and decreases and trying to increases.

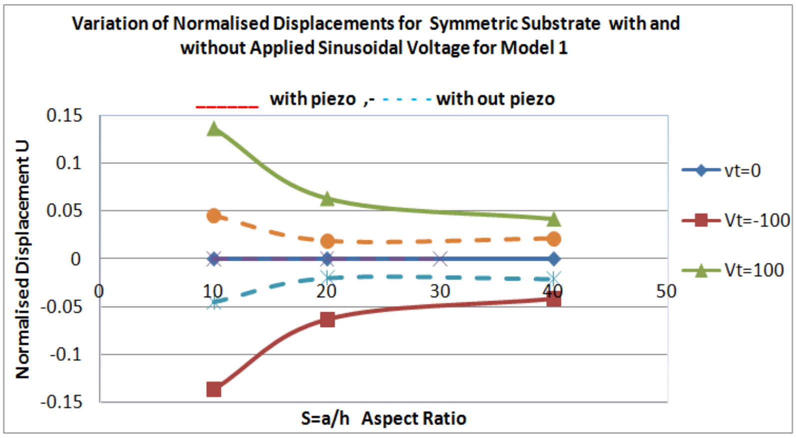

Figure 2

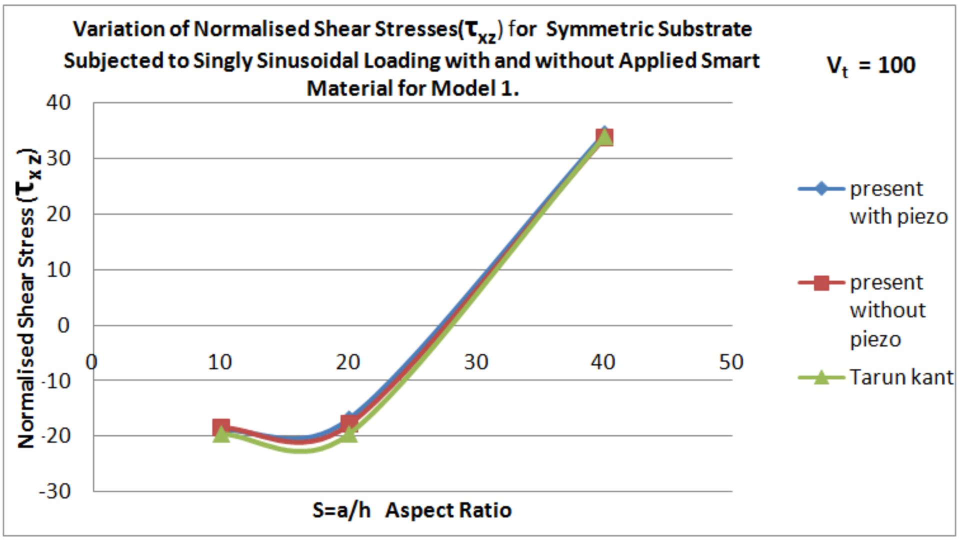

Figure 3

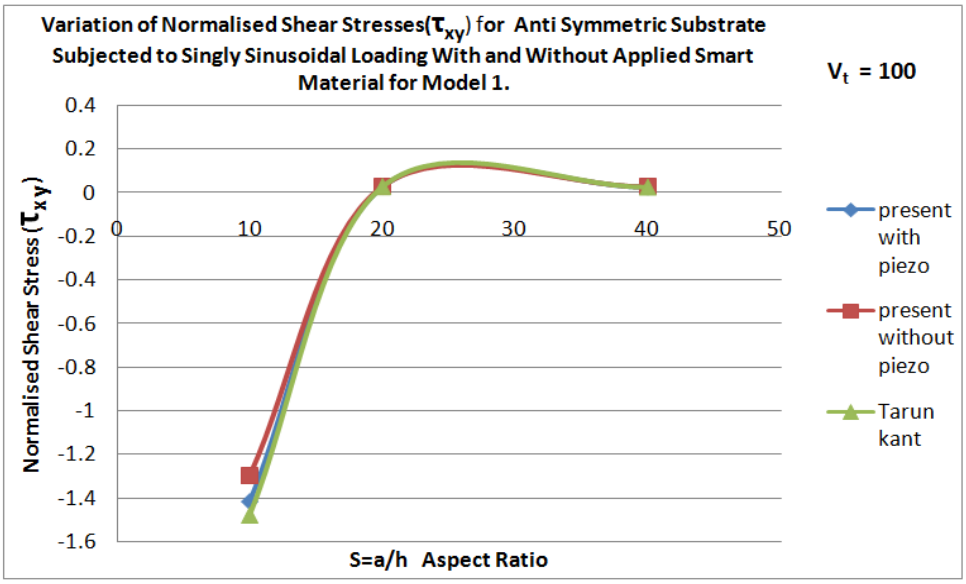

Figure 4

4. CONCLUSIONS

At very first time the present paper gives the in detail solutions for two models M1 & M2 for the composite laminated plates integrated with piezoelectric actuator under electro mechanical loading. A higher order theory is developed separately for both models M1 & M2 and the solutions are obtained with the Navier's technique. The Non dimensionalized numerical results obtained for induced in plane displacements and stresses under different electrical voltages. It is noticed that the thick laminates are more responded for actuating effects as compared with thin laminates. The basic response of piezoelectric composite laminated (smart) plates is same as that of without piezoelectric composite laminated plates. The present results concluded that higher order shear deformation theory predicts results very close to that of exact solution for smart unidirectional plates especially for transverse shear stresses and the error is approximately 4.48%. It is observed that from the investigation (for M-1), the elastic layer experiences high transverse shear stress (91.489 for anti symmetric and 21.12 for symmetric substrates at Vt=100, S=10) at top interface, when it is coupled with piezoelectric material at the top. The scope of future work is the proposed higher order theory kinematic hypothesis can be apply to pursue the investigation of transient, vibration, buckling and hygro-thermal analysis of smart laminates without any hesitation. Also the proposed theory can be applicable for shells and cylindrical vessels.

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APPENDIX

Table 1: Normalized Displacements and Stresses for Symmetric Substrate ($0^0/90^0/90^0/0^0$) with and without Applied Sinusoidal Electrical Voltages at the Top of PFRC Actuator Surface with and without Piezo Effect Model-I

Theory	With Piezo Actuator									Without Piezo Actuator								
	S=10			S=20			S=40			S=10			S=20			S=40		
	Vt=0	Vt=100	Vt=-100	Vt=0	Vt=100	Vt=-100	Vt=0	Vt=100	Vt=-100	Vt=0	Vt=100	Vt=-100	Vt=0	Vt=100	Vt=-100	Vt=0	Vt=100	Vt=-100
$\bar{u}\left(0,\frac{b}{2}\right)P$	0	-0.0631	0.116	0	-0.0631	0.136	0	-0.0416	0.0416	0	-0.02	0.0185	0	-0.045	0.045	0	-0.021	0.021
T.Kant	0.002	-0.117	0.117	0.000	-0.117	0.117	0.0001	-0.045	0.045	0.0001	-0.021	0.021	0.0001	-0.045	0.045	0.0001	-0.021	0.021
$\bar{w}\left(\frac{a}{2},\frac{b}{2}\right)P$	12.01	-9.923	33.95	13.05	-8.923	36.97	-0.239	1.486	-1.59	-0.041	0.159	-0.209	-0.267	10226	-1.76	-0.047	0.138	-0.232
T.Kant	12.18	-10345	35.67		-10.345	35.67	-0.2764	1.2454	-1.798	-0.0475	0.139	-0.2364	-0.2764	1.2454	-1.798	-0.0475	0.1395	-0.236
$\overline{\tau_{xy}}(0,0)P$	0.6181	-.958	2.194	0.886	-0.895	2.789	-0.012	-0.029	0.0003	-0.002	-0.02	0.0193	-0.016	-0.033	0.0005	-0.003	-0.027	0.0213
T.Kant	0.7536	-0.975	2.256	0.754	-0.975	2.256	-0.0262	-0.0335	0.0005	-0.090	-0.03	0.0213	-0.026	-0.0335	0.0005	-0.0902	-0.0273	0.0213
$\overline{\tau_{xz}}\left(0,\frac{b}{2}\right)P$	-21.102	21.12	-18.9	-15.4	23.12	-16.94	2.93	-28.19	34.51	2.945	-13.1	18.94	1.759	-30.19	33.709	1.201	-15.41	17.81
T.Kant	-21.354	21.34	-19.5	-21.4	21.34	-19.54	1.767	-30.74	33.97	1.216	-15.6	17.97	1076	-30.74	33.97	1.216	-15.58	17.97
$\overline{\tau_{yz}}\left(\frac{a}{2},0\right)P$	-17.897	16.229	-56.2	-14.4	17.189	-54.91	3.932	-7.9594	13.927	2.111	-3.82	6.986	1.894	-8.779	12.56	1.361	-4.02	6.743
T. Kant	-20.653	16.453	-59.5	-20.7	16.453	-59.54	1.9132	-8.876	12.679	1.3756	-4.025	6.7734	1.9132	-8.876	12.679	1.3756	-4.0256	6.7734

Table 2: Normalized Displacements and Stresses for Anti-Symmetric Substrate ($0^0/90^0/0^0/90^0$) with and without Applied Sinusoidal Electrical Voltages at the Top of PFRC Actuator Surface with and without Piezo Effect Model-I

Theory	With Piezo Actuator									Without Piezo Actuator								
	S=10			S=20			S=40			S=10			S=20			S=40		
	Vt=0	Vt=100	Vt=-100	Vt=0	Vt=100	Vt=-100	Vt=0	Vt=100	Vt=-100	Vt=0	Vt=100	Vt=-100	Vt=0	Vt=100	Vt=-100	Vt=0	Vt=100	Vt=-100
$\bar{u}\left(0,\frac{b}{2}\right)P$	0.012	-0.62	0.19	0.0005	-0.044	0.045	0.0001	-0.0204	0.0203	0.0153	-0.518	0.2034	0.00069	-0.0394	0.04299	0.00014	-0.02188	0.02761
T.Kant	0.012	-0.645	0.20	0.00055	-0.0446	0.0453	0.00012	-0.0204	0.02063	0.0129	-0.645	0.2034	0.00055	-0.0446	0.04532	0.00011	-0.0204	0.0206
$\bar{w}\left(\frac{a}{2},\frac{b}{2}\right)P$	-4.33	24.926	-33.5	-0.15	0.7047	-1.01	-0.03	0.0675	-0.12	-4.19	25.626	-32.9	-0.13	0.7397	-0.98	-0.029	0.0685	-0.11
T. Kant	-5.43	26.231	-35.6	-0.55	0.7054	-1.02	-0.01	0.0684	-0.12	-5.43	26.231	-35.6	-0.55	0.7054	-1.02	-0.190	0.0684	-0.12
$\overline{\tau_{xy}}(0,0)P$	-0.231	0.6545	-1.41	-0.009	-0.0475	0.027	-0.002	-0.0293	0.0253	-0.217	0.9869	-1.29	-0.007	-0.0412	0.02983	-0.0019	-0.0267	0.0276
T.Kant	-0.238	0.9856	-1.47	-0.010	-0.0488	0.0280	-0.002	-0.03	0.0255	-0.238	0.9856	-1.47	-0.010	-0.0488	0.02801	-0.0020	-0.03000	0.0255
$\overline{\tau_{xz}}\left(0,\frac{b}{2}\right)P$	6.654	-91.489	104.79	0.9720	-32.303	34.25	0.762	-16.47	17.997	6.928	-89.689	106.28	0.9960	-31.390	35.387	0.7982	-15.9632	18.378
T. Kant	6.899	-95.749	110.10	0.9923	-32.547	34.35	0.763	-16.5	18.021	6.899	-95.749	110.101	0.9923	-32.547	34.352	0.7632	-16.501	18.021
$\overline{\tau_{yz}}\left(\frac{a}{2},0\right)P$	8.207	-47.478	63.8	1.324	-6.0926	8.74	1.071	-2.422	4.56	8.765	-44.864	64.4	1.579	-5.9961	8.91	1.1902	-2.349	4.7123
T. Kant	8.388	-48.311	65.6	1.346	-6.1998	8.82	1.078	-2.45	4.58	8.388	-48.311	65.6	1.346	-6.1998	8.82	1.0798	-2.4532	4.5832